Unit 3: Production and Cost

Name: ______________________

Date: __/__/__

Production Function

The production function of a firm is a relationship between inputs used and output produced by the firm. For various quantities of inputs used, it gives the maximum quantity of output that can be produced. The inputs that a firm uses in the production process are called factors of production.

To produce output, a firm may require any number of different inputs. However, for the time being, here we consider a firm that produces output using only two factors of production – factor 1 and factor 2. Our production function, therefore, tells us what maximum quantity of output can be produced by using different combinations of these two factors. We may write the production function as

$$q = f(x_1 - x_2)$$

It says that by using x_1 amount of factor 1 and x_2 amount of factor 2, we can at most produce q amount of the commodity

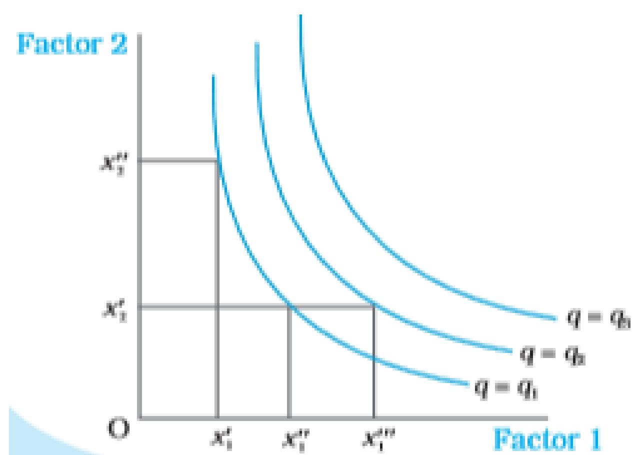
Isoquant

It is just an alternative way of representing the production function. Consider a production function with two inputs factor 1 and factor 2. An isoquant is the set of all possible combinations of the two inputs that yield the same maximum possible level of output. Each isoquant represents a level of output and is labelled with that amount of output.

In the diagram, we have three isoquants for the three output levels, namely $q = q_1$, $q = q_2$ and $q = q_3$ in the inputs plane. Two input combinations (x'_1, x''_2) and (x''_1, x'_2) give us the same level of output q_1.

If we fix factor 2 at x'_2 and increase factor 1 to x'''_1, output increases and we reach a higher isoquant, $q = q_2$.

When marginal products are positive, with greater amount of one input, the same level of output can be produced by using lesser amount of the other. Therefore, isoquants are negatively sloped.



Short Run and Long Run

Before we begin with any further analysis, it is important to discuss two concepts– the short run and the long run. In the short run, a firm cannot vary all the inputs. One of the factors – factor 1 or factor 2 – cannot be varied, and therefore, remain fixed in the short run. To vary the output level, the firm can vary only the other factor.

The factor that remains fixed is called the fixed input whereas the other factor which the firm can vary is called the variable input. Consider the example represented through the table below.

Suppose, in the short run, factor 2 remains fixed at 5 units. Then the corresponding column shows the different levels of output that the firm may produce using different quantities of factor 1 in the short run.

In the long run, all factors of production can be varied. A firm to produce different levels of output in the long run may vary both the inputs simultaneously. So, in the long run, there is no fixed input.

Factor		**x_2**						
		0	**1**	**2**	**3**	**4**	**5**	**6**
x_1	**0**	0	1	2	3	4	5	6
	1	0	1	3	7	10	12	13
	2	0	3	10	18	24	29	33
	3	0	7	18	30	40	46	50
	4	0	10	24	40	50	56	57
	5	0	12	29	46	56	58	59
	6	0	13	33	50	57	59	60

Total Product, Average Product and Marginal Product

Total Product

Total Product Suppose we vary a single input and keep all other inputs constant. Then for different levels of employment of that input, we get different levels of output from the production function. This relationship between the variable input and output, keeping all other inputs constant, is often referred to as Total Product (TP) of the variable input.

If we keep the second factor constant and vary the first factor, we get the following value for q

$$q = f(x_1 : \bar{x}_2)$$

Let us again look at the table. Suppose factor 2 is fixed at 4 units. Now in the table, we look at the column where factor 2 takes the value 4. As we move down along the column, we get the output values for different values of factor 1. This is the total product of factor 1 schedule with $x_2 = 4$. At $x_1 = 0$, the TP is 0, at $x_1 = 1$, TP is 10 units of output, at $x_1 = 2$, TP is 24 units of output and so on. This is also sometimes called total return to or total physical product of the variable input.

Average Product

Average product is defined as the output per unit of variable input. We calculate it as

$$AP_1 = \frac{TP}{x_1} = \frac{f(x_1 - \bar{x}_2)}{x_1}$$

In the table, we have already seen the total product of factor 1 for $x_2 = 4$. In the table we reproduce the total product schedule and extend the table to show the corresponding values of average product and marginal product. The first column shows the amount of factor 1 and in the fourth column we get the corresponding average product value. It shows that at 1 unit of factor 1, AP_1 is 10 units of output, at 2 units of factor 1, AP_1 is 12 units of output and so on.

Marginal Product

Marginal product of an input is defined as the change in output per unit of change in the input when all other inputs are held constant. When factor 2 is held constant, the marginal product of factor 1 is

$$MP_1 = \frac{change\ in\ output}{change\ in\ input}$$

$$= \frac{\Delta q}{\Delta x_1}$$

For example

If the input changes by discrete units, the marginal product can be defined in the following way. Suppose, factor 2 is fixed at $\bar{x}_2$. With $\bar{x}_2$ amount of factor 2, let, according to the total product curve, x_1 units of factor 1 produce 20 units of the output and $x_1 - 1$ units of factor 1 produce 15 units of the output. We say that the marginal product of the x_1th unit of factor 1 is

$$MP_1 = f(x_1 : \bar{x}_2) - f(x_1 - 1 : \bar{x}_2)$$

$$= TP\ at\ x_1 units - (TP\ at\ x_1 - 1\ unit)$$

$$= (20 - 15) units\ of\ output$$

$$= 5\ units\ of\ output$$

Since inputs cannot take negative values, marginal product is undefined at zero level of input employment. Marginal products are additions to total product. For any level of employment of an input, the sum of marginal products of every unit of that input up to that level gives the total product of that input at that employment level. So total product is the sum of marginal products.

In the example represented through above table, if we keep factor 2 constant say, at 4 units, we get a total product schedule. From the total product, we then derive the marginal product and average product of factor 1. The third column shows that at zero unit of factor 1, MP_1 is undefined. At $x_1 = 1$, Mp_1 is 10 units of output, at x1 = 2, MP_1 is 14 units of output and so on

Factor 1	**TP**	**MP1**	**AP1**
0	0	-	-
1	10	10	10
2	24	14	12
3	40	16	13.33
4	50	10	12.5
5	56	6	11.2
6	57	1	95

The Law Of Diminishing Marginal Product And The Law Of Variable Proportions

The law of diminishing marginal product says that if we keep increasing the employment of an input, with other inputs fixed, eventually a point will be reached after which the resulting addition to output (i.e., marginal product of that input) will start falling.

A somewhat related concept with the law of diminishing marginal product is the law of variable proportions. It says that the marginal product of a factor input initially rises with its employment level. But after reaching a certain level of employment, it starts falling.

The reason behind the law of diminishing returns or the law of variable proportion is the following. As we hold one factor input fixed and keep increasing the other, the factor proportions change. Initially, as we increase the amount of the variable input, the factor proportions become increasingly suitable for the production and marginal product increases.

But after a certain level of employment, the production process becomes too crowded with the variable input and the factor proportions become less and less suitable for the production. It is from this point that the marginal product of the variable input starts falling.

Let us look at the table below with factor 2 fixed at 4 units, the table shows us the TP, MP_1 and AP_1 for different values of factor 1. We see that up to the employment level of 3 units of factor 1, its marginal product increases. Then it starts falling.

Factor 1	**TP**	**MP_1**	**AP_1**
0	0	-	-
1	10	10	10
2	24	14	12
3	40	16	13.33
4	50	10	12.5
5	56	6	11.2
6	57	1	95

Shapes of TP, AP and MP Curves

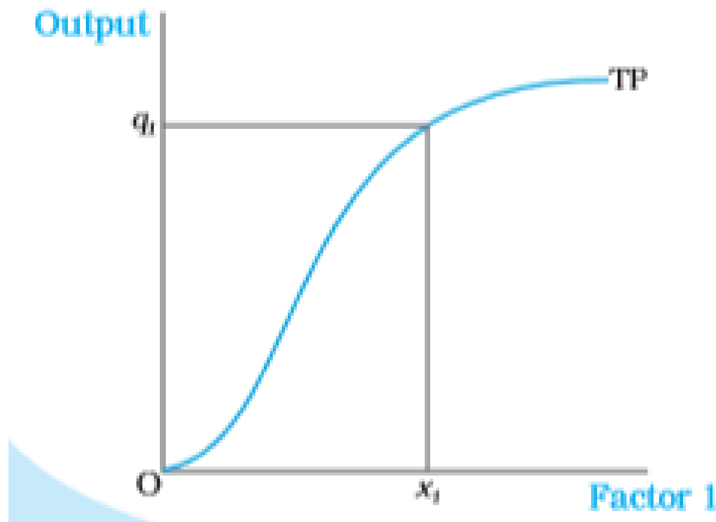
An increase in the amount of one of the inputs keeping all other inputs constant generally results in an increase in output. The table above shows how the total product changes as the amount of factor 1 increases. The total product curve in the input-output plane is a positively sloped curve.

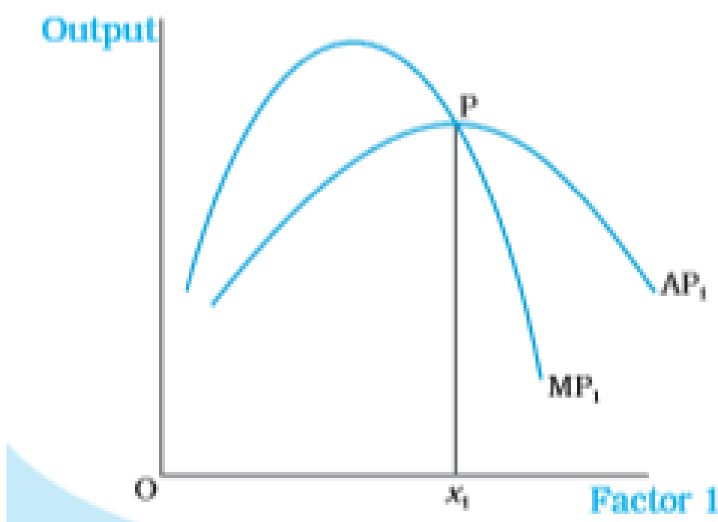
The diagram shows the shape of the total product curve for a typical firm. We measure units of factor 1 along the horizontal axis and output along the vertical axis. With x_1 units of factor 1, the firm can at most produce q_1 units of output. According to the law of variable proportions, the marginal product of an input initially rises and then after a certain level of employment, it starts falling. The MP curve in the input-output plane, therefore, looks like an inverse 'U'-shaped curve.

For the first unit of the variable input, one can easily check that the MP and the AP are same. Now as we increase the amount of input, the MP rises. AP being the average of marginal products, also rises, but rises less than MP. Then, after a point, the MP starts falling.

However, if the value of MP remains higher than the value of the prevailing AP, the latter continues to rise. Once MP has fallen sufficiently, its value becomes less than the prevailing AP and the latter also starts falling. So AP curve is also inverse 'U'-shaped.

If the AP increases, it must be the case that MP is greater than AP. Otherwise, AP cannot rise. Similarly, when AP falls, MP must be less than AP. It, therefore, follows that MP curve cuts AP curve from above at its maximum. The diagram shows the shapes of AP and MP curves for a typical firm. The AP of factor 1 is maximum at x_1. To the left of x_1, AP is rising, and MP is greater than AP. To the right of x_1, AP is falling, and MP is less than AP.





Returns to Scale

So far we looked at various aspects of production function when a single input varied and others remained fixed. Now we shall see what happens when all inputs vary simultaneously.

1. Constant returns to scale (CRS) is a property of production function that holds when a proportional increase in all inputs results in an increase in output by the same proportion.
2. Increasing returns to scale (IRS) holds when a proportional increase in all inputs results in an increase in output by more than the proportion.
3. Decreasing returns to scale (DRS) holds when a proportional increase in all inputs results in an increase in output by less than the proportion.

For example, suppose in a production process, all inputs get doubled. As a result, if the output gets doubled, the production function exhibits CRS. If output is less than doubled, the DRS holds, and if it is more than doubled, the IRS holds.

Costs

Short Run Costs

We have previously discussed the short run and the long run. In the short run, some of the factors of production cannot be varied, and therefore, remain fixed. The cost that a firm incurs to employ these fixed inputs is called the total fixed cost (TFC). Accordingly, the cost that a firm incurs to employ these variable inputs is called the total variable cost (TVC). Adding the fixed and the variable costs, we get the total cost (TC) of a firm

$$TC = TVC + TFC$$

The short run average cost (SAC) incurred by the firm is defined as the total cost per unit of output. We calculate it as

$$SAC = \frac{TC}{q}$$

The short run marginal cost (SMC) is defined as the change in total cost per unit of change in output

$$SMC = \frac{change\ in\ total\ cost}{change\ in\ output} = \frac{\Delta TC}{\Delta q}$$

The average fixed cost (AFC) is defined as the total variable cost per unit of output. We calculate it as

$$AFC = \frac{TFC}{q}$$

Hence

$$SAC = AVC + AFC$$

The average variable cost (AVC) is defined as the total variable cost per unit of output. We calculate it as

$$AVC = \frac{TVC}{q}$$

Long Run Costs

Overall, all inputs are variable. Long run average cost (LRAC) is defined as cost per unit of output, i.e.

$$LRAC = \frac{TC}{q}$$

Long run marginal cost (LRMC) is the change in total cost per unit of change in output. When output changes in discrete units, then, if we increase production from q_1–1 to q_1 units of output, the marginal cost of producing q_1^{th} unit will be measured as

$$LRMC = (TC\ at\ q_1 units) - (TC\ at\ q_1 - 1\ units)$$